

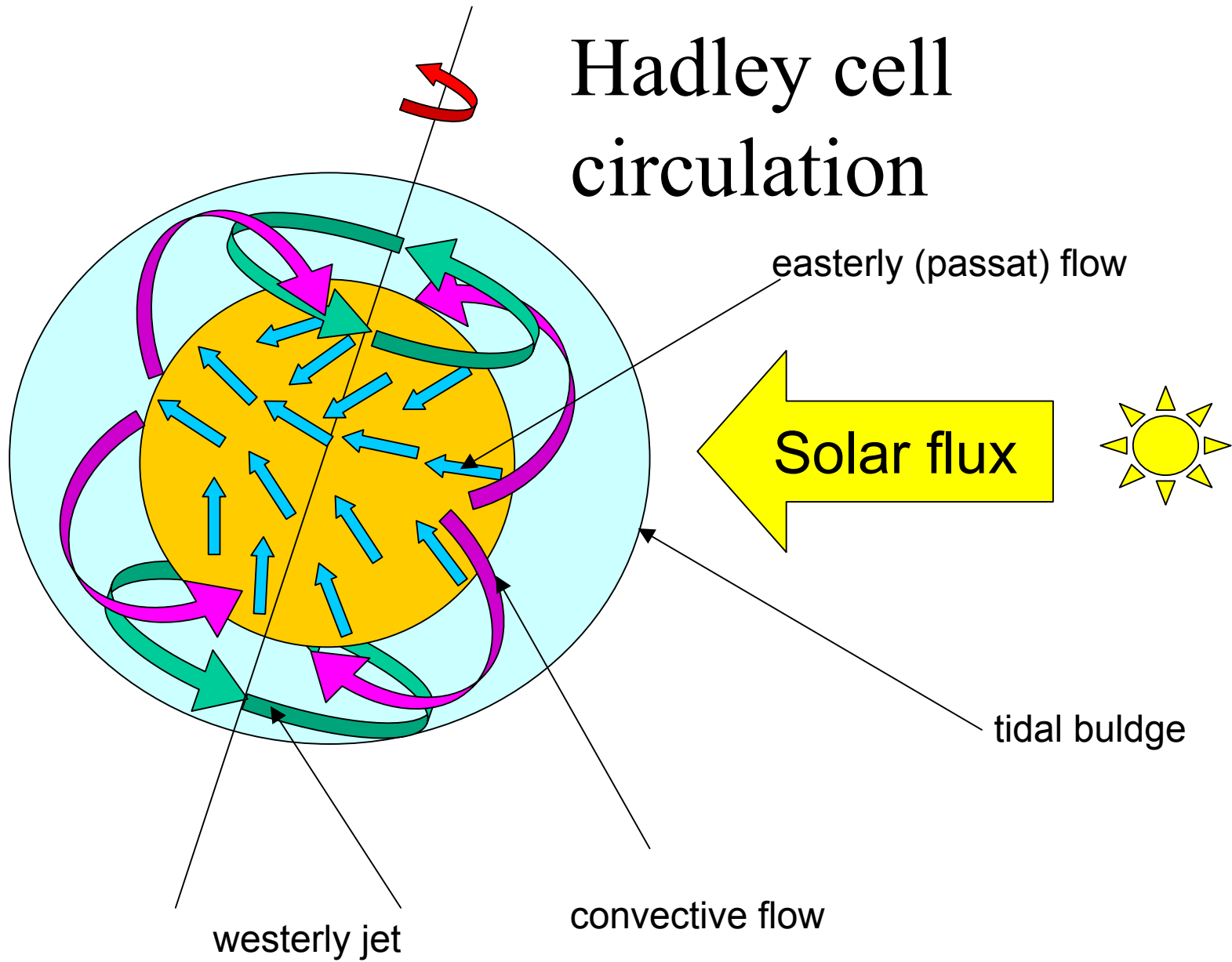
The dynamics of planetary atmospheres

Theoretical concept is formulated in the

Geophysical Fluid Dynamics –

the dynamics of stratified fluid on a rotating sphere

Hadley cell circulation



“Primitive” equations: fluid on a rotating sphere

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\alpha \nabla p + \nu \nabla^2 \mathbf{v} - 2\boldsymbol{\omega} \times \mathbf{v} + \mathbf{g},$$

momentum

$$C_V \frac{dT}{dt} + p \frac{d\alpha}{dt} = q + f,$$

energy

$$\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0,$$

continuity

$$p = \rho RT.$$

eqn. of state

modifications

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla \left(\frac{1}{2} |\mathbf{v}|^2 + \frac{p}{\rho} + \Phi \right) + (\boldsymbol{\Omega} + 2\boldsymbol{\omega}) \times \mathbf{v} = \nu \nabla^2 \mathbf{v}$$

$$\nabla \Phi = -\mathbf{g}, \Phi - \text{geopotential}$$

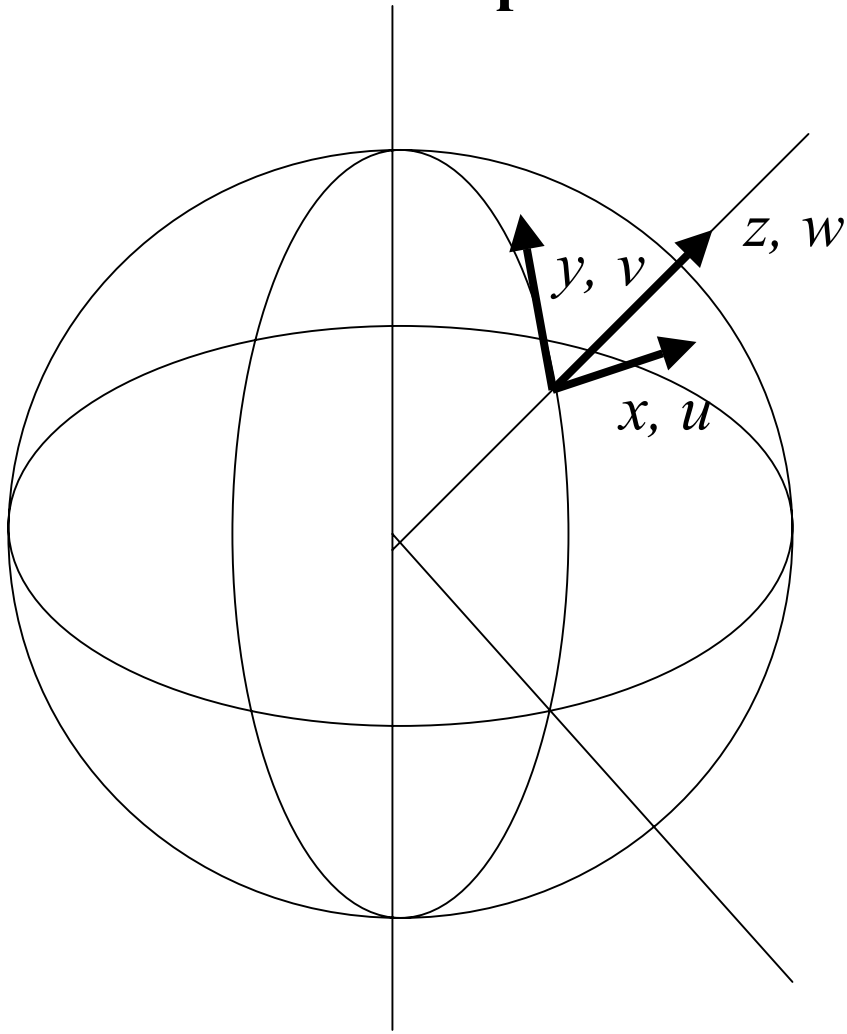
$$(\boldsymbol{\Omega} + 2\boldsymbol{\omega}), \boldsymbol{\Omega} = \text{rot } \mathbf{v} \quad \text{absolute vorticity}$$

Boussinesq approximation:

$$\frac{d\rho}{dt} = 0 \quad \text{everywhere but gravity}$$

$$\frac{d\mathbf{v}}{dt} = -\nabla p - 2\rho \boldsymbol{\omega} \times \mathbf{v} + \rho \mathbf{g}$$

Spherical coordinate system:



$$\frac{du}{dt} = -\alpha \frac{\partial p}{\partial x} - 2\omega(w \sin \theta - v \cos \theta)$$

$$\frac{dv}{dt} = -\alpha \frac{\partial p}{\partial y} - 2\omega u \cos \theta$$

$$\frac{dw}{dt} = -g - \alpha \frac{\partial p}{\partial z} + 2\omega u \cos \theta$$

Useful numbers

$$f = 2\omega \cos \theta = 2\omega \sin \varphi \quad \text{Coriolis parameter}$$

$$f \approx 2\omega \sin \varphi_0 + \frac{2\omega}{R} \cos \varphi_0 y \quad \beta\text{-plane approximation}$$

$$\beta = \frac{2\omega}{R} \cos \varphi_0 \quad \text{Rossby parameter}$$

$$\text{Rossby number} \quad \text{Ro} = \frac{V}{fL}$$

Adjustments

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + w \frac{\partial \mathbf{u}}{\partial z} + f \mathbf{k} \times \mathbf{u} = -\frac{\nabla p}{\rho} \quad \text{equation of horizontal motion}$$

$$f \mathbf{k} \times \mathbf{u} \approx -\frac{\nabla p}{\rho},$$

$$\mathbf{u}_g = \frac{1}{\rho f} \mathbf{k} \times \nabla p$$

geostrophic approximation

$$\frac{dw}{dt} + fu \operatorname{ctg} \varphi + g = -\alpha \frac{\partial p}{\partial z},$$

$$g \approx -\alpha \frac{\partial p}{\partial z}.$$

hydrostatic approximation

thermal wind equation

$$\frac{\partial}{\partial z} \mathbf{u}_g = \frac{R}{f\rho} \left(\frac{\partial T}{\partial z} \mathbf{k} \times \nabla p - \frac{\partial p}{\partial z} \mathbf{k} \times \nabla T \right) \approx \frac{g}{fT} \frac{\partial p}{\partial z} \mathbf{k} \times \nabla T$$

Opposite case: $Ro \gg 1$

$$u_c \approx \left(-\alpha R_c \frac{\partial p}{\partial n} \right)^{1/2}.$$

cyclostrophic approximation

Wave motion in the atmosphere

- Elasticity sound
- Buoyancy gravity waves
- Coriolis force inertial waves

Planetary-scale forced oscillations of any type
are called tides

acoustic waves

$$\frac{d\mathbf{v}}{dt} = -\alpha \nabla p, \quad \frac{dp}{dt} = -\gamma p \nabla \mathbf{v}$$

$$p_0 = \text{const}, \quad \alpha_0 = \text{const}, \quad \mathbf{v}_0 = 0$$

$$\frac{\partial^2 p}{dt^2} = -\gamma \alpha_0 p_0 \nabla^2 p.$$

if $u_0 \neq 0$,

$$\left(\frac{\partial}{dt} + u_0 \frac{\partial}{dx} \right)^2 p = -\gamma \alpha_0 p_0 \nabla^2 p,$$

$$c = u_0 \frac{k_x}{k} \pm \sqrt{\gamma \alpha_0 p_0}.$$

gravity waves

$$\frac{d^2 z}{dt^2} = g \frac{\theta - \theta_0}{\theta_0},$$

θ -potential temperature

$$\theta(z) \approx \theta_0(0) + z \frac{d\theta}{dz}.$$

$$\frac{d^2 z}{dt^2} + \frac{g}{\theta} \frac{d\theta}{dz} z = 0.$$

$$\omega_g^2 = \frac{g}{\theta} \frac{d\theta}{dz}$$

Brunt-Vajsala frequency

For non-vertically propagating waves, $\omega^2 = \omega_g^2 \frac{k_H^2}{k_V^2}$

Group velocity $c_g = \frac{\omega_g}{k_V^2} (-\mathbf{i}k_V + \mathbf{k}k_H).$

Rossby waves

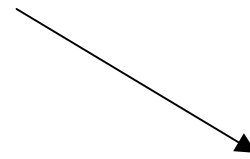
$$\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} - fv + \frac{\partial p}{\partial x} \rho = 0$$

$$\frac{\partial v}{\partial t} + U \frac{\partial v}{\partial x} + fv + \frac{\partial p}{\partial y} \rho = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

Search for the solution in form

$$\frac{u}{\tilde{u}(y)} = \frac{v}{\tilde{v}(y)} = \frac{P}{\tilde{P}(y)} = A e^{i(\omega t + kx)}.$$


$$i(\omega + kU)\tilde{u} - f\tilde{v} + ik\tilde{P} = 0$$

$$i(\omega + kU)\tilde{v} + f\tilde{u} + \frac{\partial \tilde{P}}{\partial y} = 0$$

$$ik\tilde{u} + \frac{\partial \tilde{v}}{\partial y} = 0.$$

Eliminating P and use incompressibility,

$$(\omega + kU) \left(-\frac{\partial^2 \tilde{v}}{\partial y^2} + k^2 \tilde{v} \right) - k\beta \tilde{v} = 0,$$

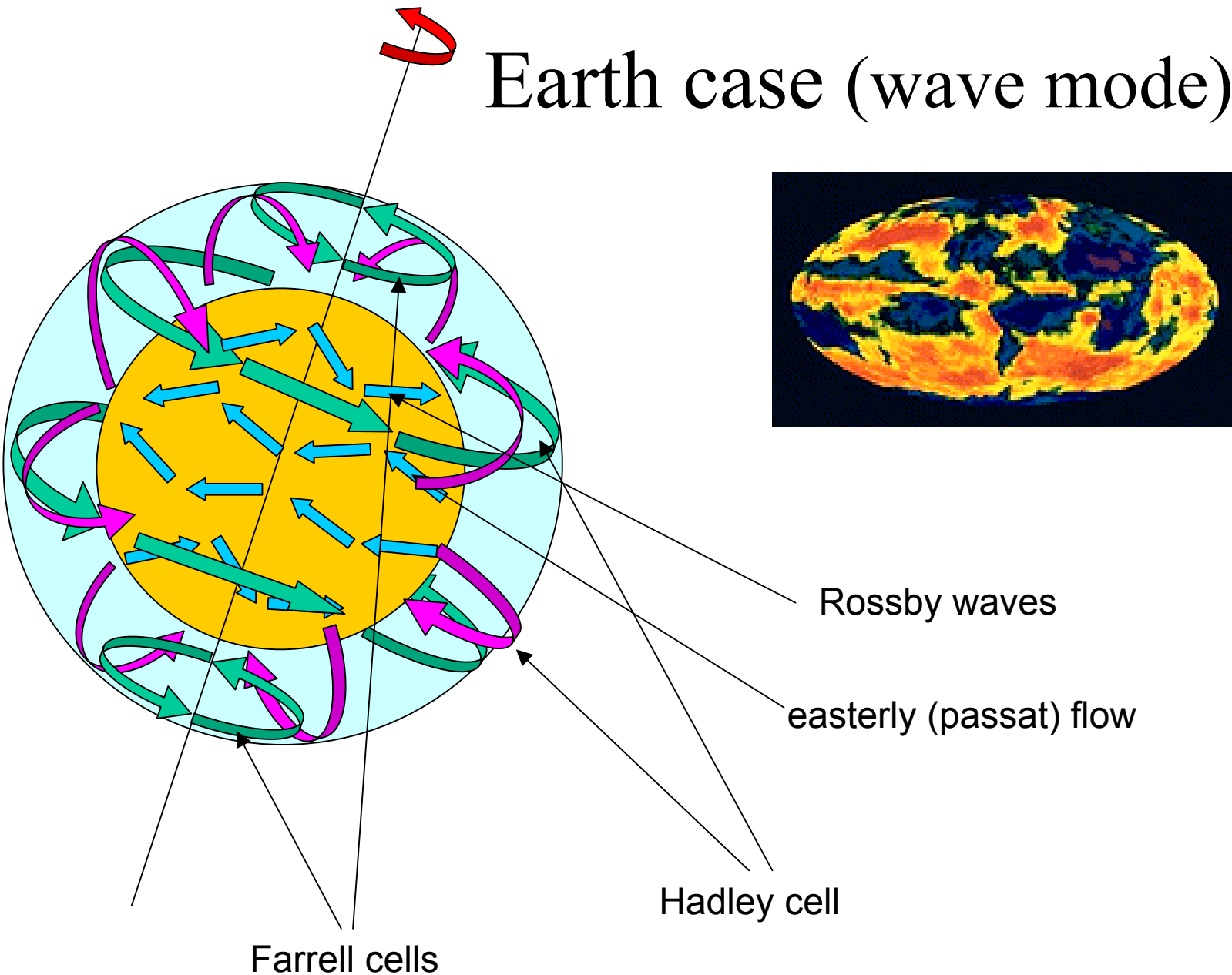
or

$$\frac{\partial^2 \tilde{v}}{\partial y^2} + \left(\frac{\beta}{U - c} - k^2 \right) \tilde{v} = 0,$$

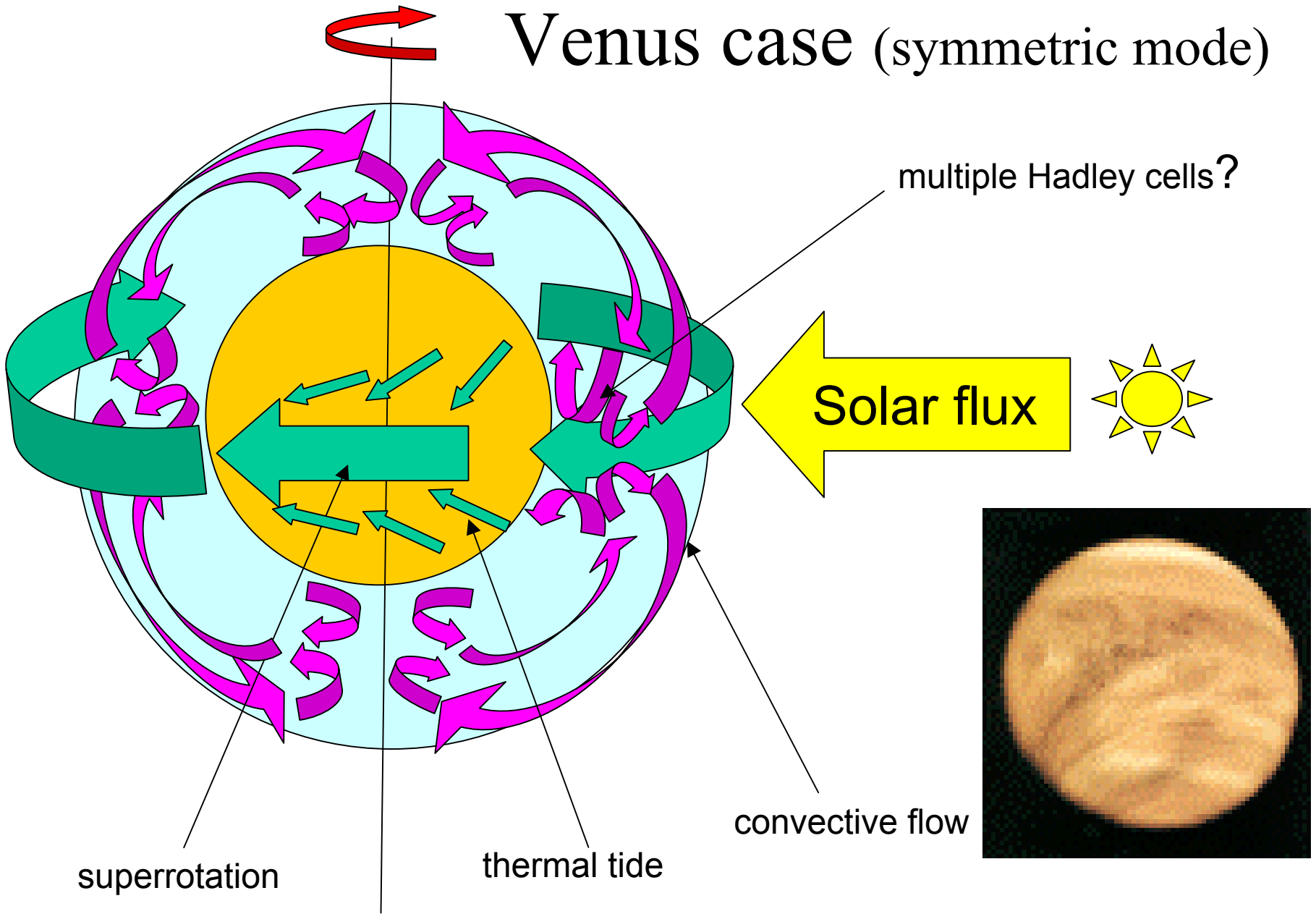
general solution is

$$\tilde{v} = C_1 \cos \left[\left(\frac{\beta}{U - c} - k^2 \right)^{1/2} y \right] + C_2 \sin \left[\left(\frac{\beta}{U - c} - k^2 \right)^{1/2} y \right].$$

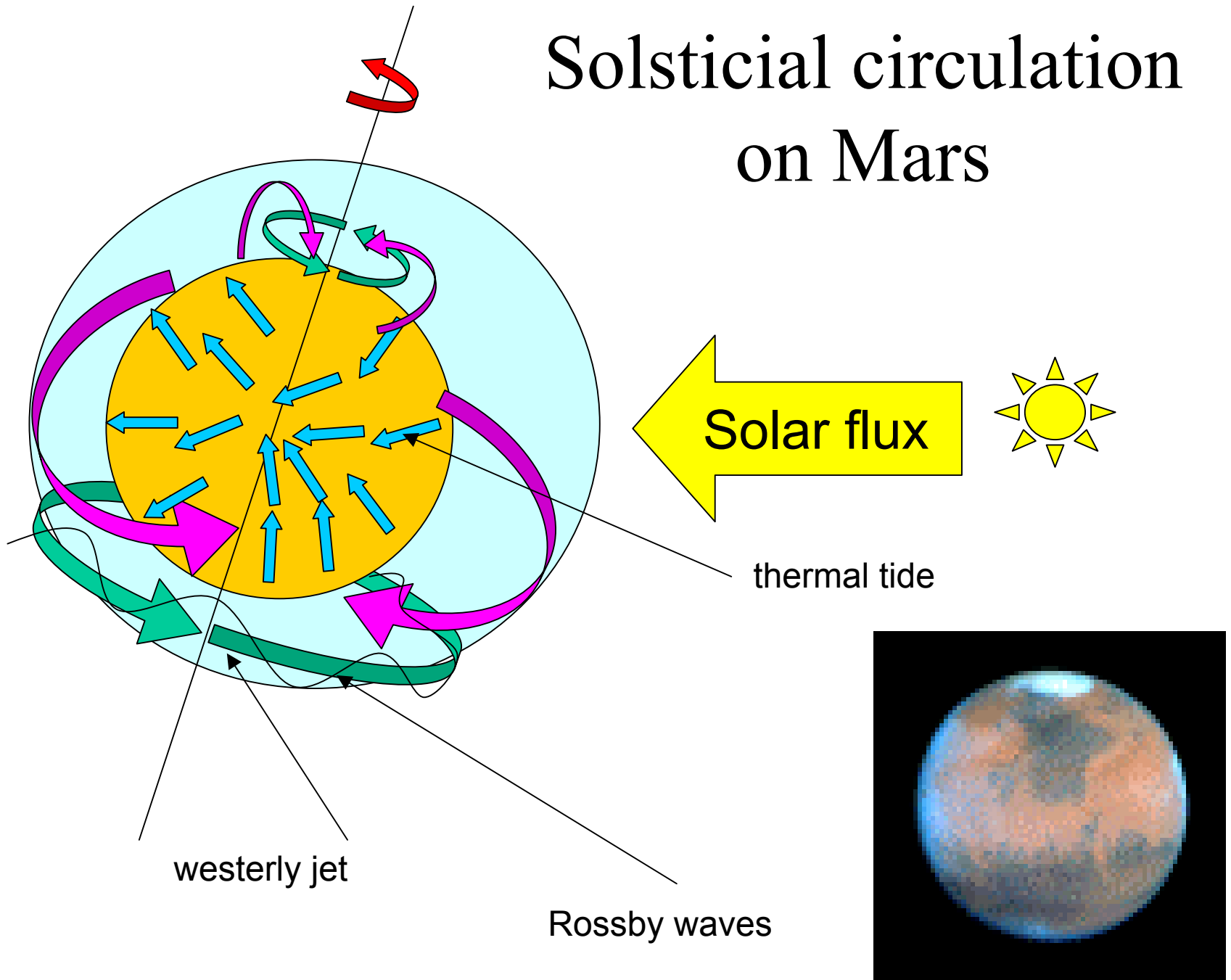
Earth case (wave mode)



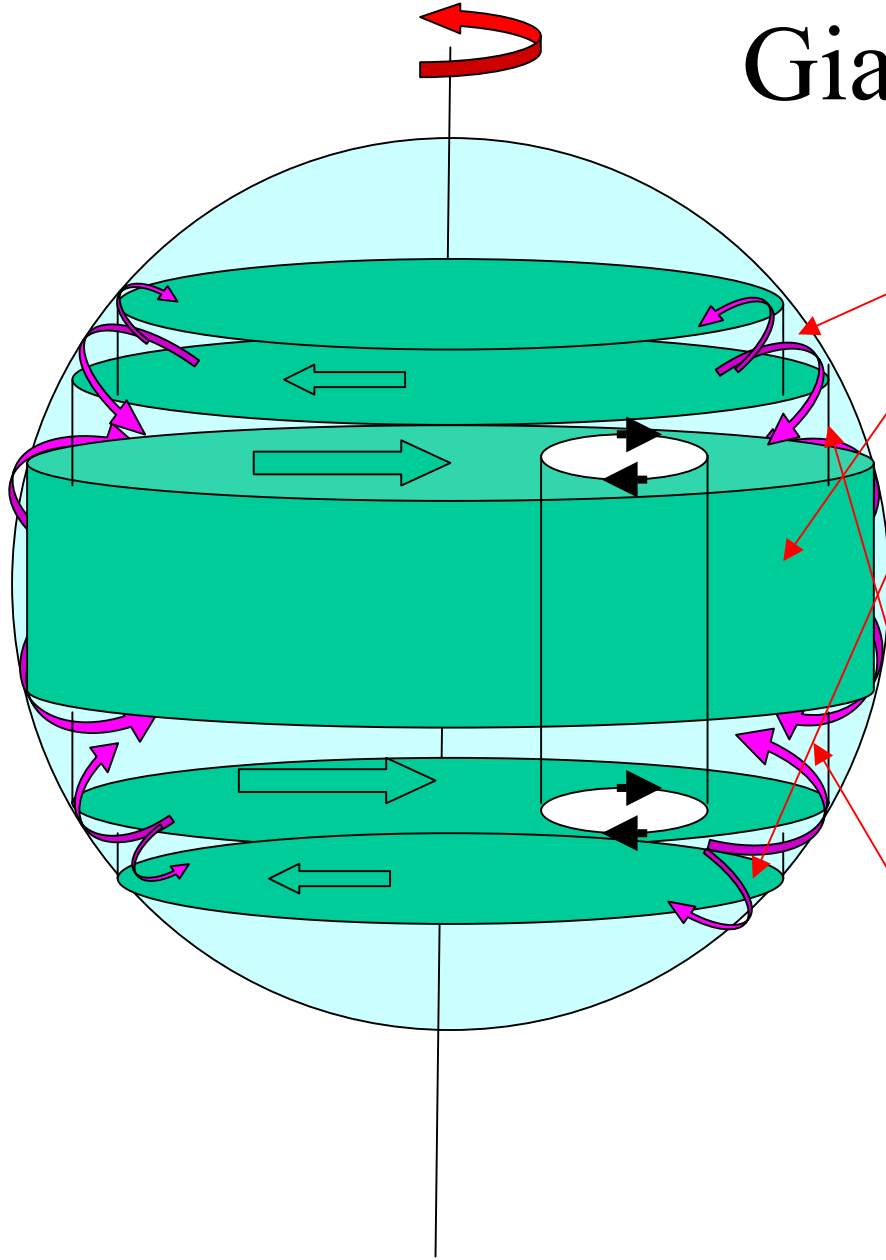
Venus case (symmetric mode)



Solstitial circulation on Mars



Giant planets



zones

belts

