

Lecture notes

"sun-planet connections"

given at MPS

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Magnetic field in electrically
conductive medium (fluid, gas)

Electric field

$$\vec{E} = \underbrace{-\vec{v} \times \vec{B}}_{\text{convective}} + \underbrace{\frac{1}{\sigma} \vec{j}}_{\text{ohmic}} \quad \dots (1)$$

$$= -\vec{v} \times \vec{B} + \frac{1}{\mu_0 \sigma} \nabla \times \vec{B} \quad \dots (2)$$

(Ampère's law)

Induction eq.

$$\partial_t \vec{B} = -\nabla \times \vec{E} \quad \dots (3)$$

Combine (2) and (3) and $\nabla \cdot \vec{B} = 0$

↘

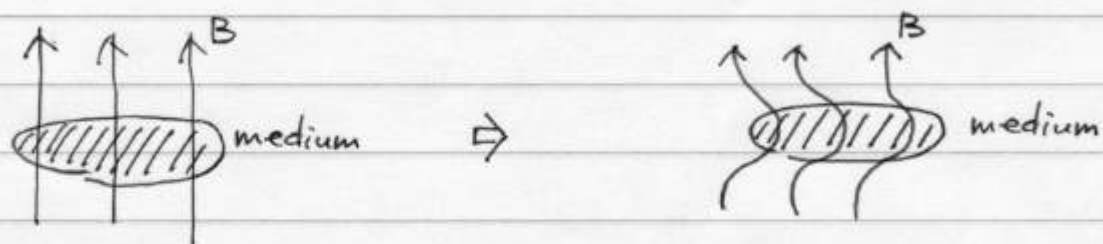
$$\partial_t \vec{B} = \nabla \times (\vec{v} \times \vec{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \vec{B} \quad \dots (4)$$

Evolution of
mag. field

(A) frozen-in

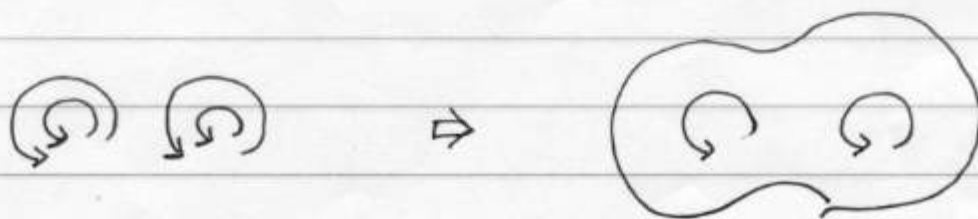
(B) diffusion

(A) Frozen-in magnetic field



Magnetic field moves with flow/medium.

(B) Diffusion



Magnetic field becomes weaker.

Space plasma ... collisionless, conductive medium.
Conductivity σ very large.

↘ Frozen-in magnetic field

Consequences

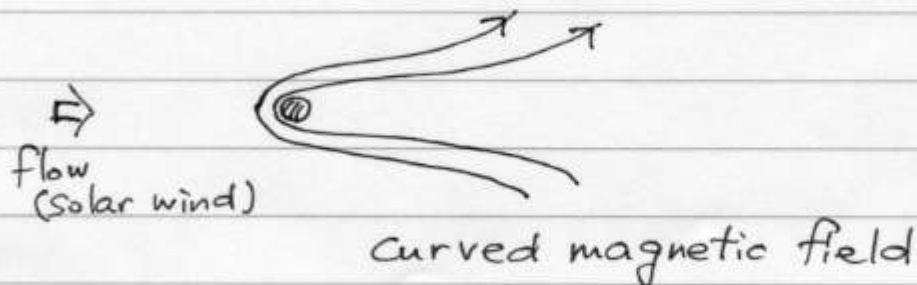
(1) Magnetic field moves with flow
(e.g. solar wind)

(2) Different media cannot be mixed.

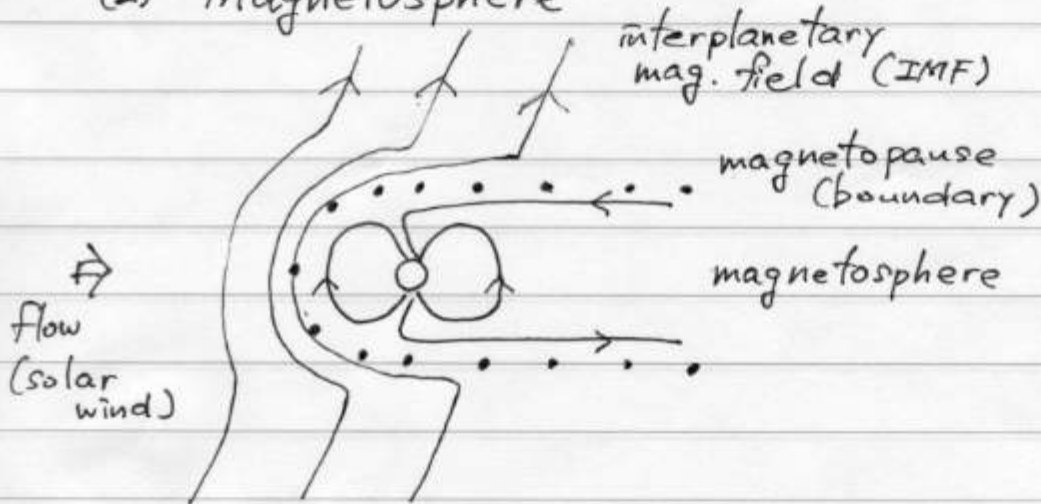
↘ Formation of boundaries
(e.g. magnetopause)

Applications

(1) Comet tail (Alfvén's idea, 1957)



(2) Magnetosphere



How large is magnetosphere?

Pressure balance

$$\frac{1}{2} \rho v^2 = \frac{B^2}{2\mu_0} \quad \dots (5)$$

dynamic pressure	=	magnetic pressure
(kinetic energy density)		(mag. energy density)

solar wind

planet

Dipole magnetic field (for planet)

$$B \propto \frac{1}{r^3}$$

$$\Rightarrow \frac{B}{B_{\text{surf}}} = \left(\frac{R_{\text{surf}}}{R_{\text{mp}}} \right)^3 \quad \dots (6)$$

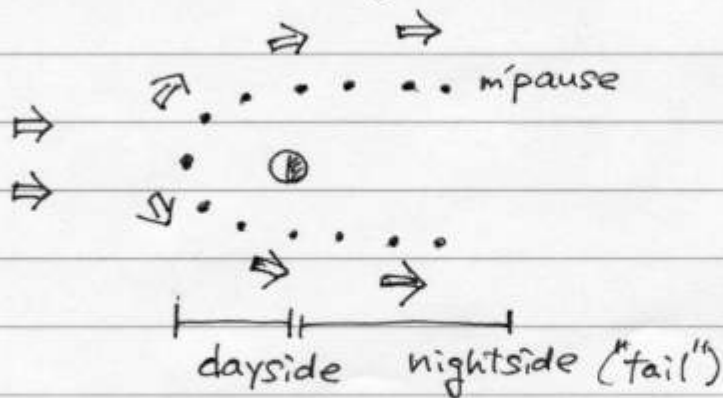
Combine (5) and (6)

$$\Rightarrow \frac{R_{\text{mp}}}{R_{\text{surf}}} = \left(\frac{B_{\text{surf}}}{\mu_0 \rho v^2} \right)^{1/6} \quad \dots (7)$$

Classification of planets

	$R_{\text{mp}} > R_{\text{surf}}$	$R_{\text{mp}} < R_{\text{surf}}$
Solid surface	Mercury	Earth Moon
gas surface (atmosphere)	Earth Jupiter, Saturn Uranus, Neptune	Venus, Mars

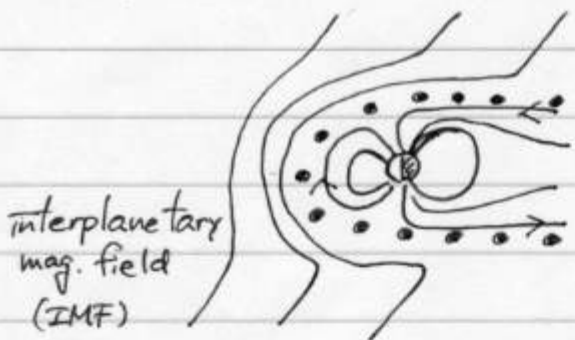
• Flow around magnetosphere



Surface mag. field 30 000 nT

Magnetopause distance ... 10 ~ 11 R_E (dayside)

• Magnetic field

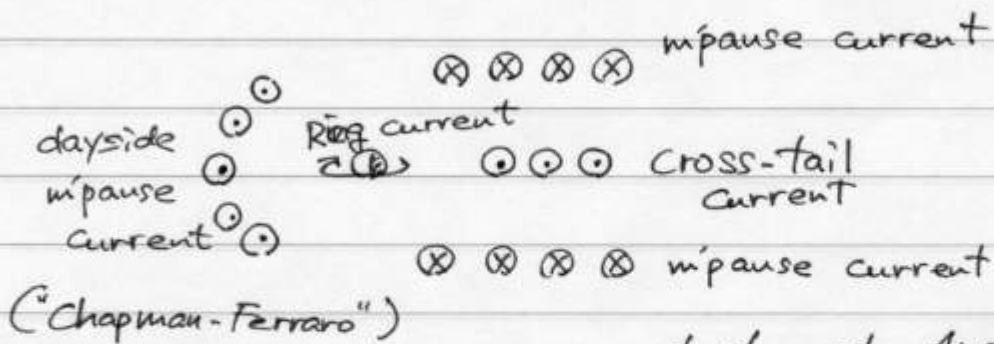


solar wind ... curved mag. field
 dayside magnetosphere ... dipolar field
 nightside ... elongated / stretched field

• Plasma populations

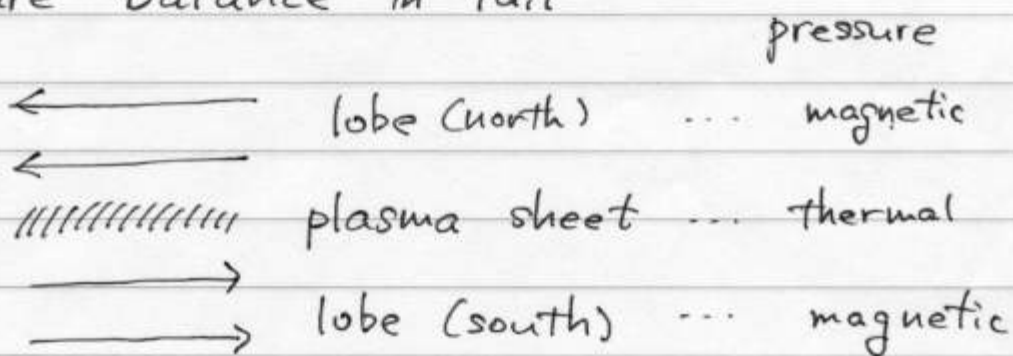


• Electric current



consistent with Ampère's law
~~discrete current~~ $\vec{j} = \frac{1}{\mu_0} \nabla \times \vec{B}$

• Pressure balance in tail



$$P_{\text{plasma sheet}} = P_{\text{magnetic lobe}}$$

• Why is tail-field stretched?

possibility 1 ... viscosity/friction

(Axford and Hines, 1961)

